A New Methodology for Modelling Stochastically the Time to Capsize

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ABSTRACT

"Design a safe ship" is the best annotation of putting human life as the foremost issue. Flooding related damage is regarded as the major contributor to risk to life, thus the phenomenon of ship vulnerability to flooding with quantifiable accuracy must be taken into account. Regarding this, a measure of "time to capsize" has been respected as a simplified manner to quantify ship survivability. Based on the latest research activities taken place at the Ship Stability Research Centre (SSRC), this paper aims to describe a new methodology for modelling the stochastic behaviour of ship time to capsize following flooding by deploying Bayesian regression techniques for selected Probit model.

Keywords: *ship survivability assessment, probit model, Bayesian regression, uncertainty quantification*

INTRODUCTION

The remarkable biggest cruise ship to date *MS Oasis of the Seas* made her debut in December, 2009, holding an excess of 8,000 passengers and crew. Evidently ship stability safety for passenger vessels is a crucial subject to naval architecture and can be hardly overlooked. Stability standards have ever been improved by continuous effort to ensure the ship attained an adequate safe design. Flooding related damage is regarded as a major and great threat to the safety of life at sea, thus a reasonable assessment of ship survivability after flooding incident commence as a marked out target is undertaken frequently and deeply. Hence the immediate questions brought forth of 1) How long it takes a vessel to capsize from the instant of hull breach given a specific extent of damage , loading and environmental conditions, and 2) How reliable is its predicted survivability, have yet to be answered satisfactorily.

In search of appropriate answers, a measure of **"time to capsize"** has been respected as a simplified way to quantify ship survivability by taking into account the related confounded physical effects. A time-based mathematical model was first proposed by SSRC, (Jasionowski, 2006), in the course of the SAFEDOR project (www.safedor.org). Correspondingly a series of dedicated physical model experiment have been undertaken through a succession of related projects partially supported by EC. The collected testing records lay a solid foundation for validating the reasoning behind the proposed models is currently on-going.

On the basis of the developed analytical model and benchmark data from survivability tests executed to date, this paper attempts to present an alternative for modelling stochastically the time necessary for a ship to capsize. The most particular attribute of this new method is to state the plausible inferences based on gathered experimental data, i.e. let the facts speak for themselves. The most direct breakthrough of this methodology is to improve the degree of common belief of the developed approach addressing survivability assessment, and to quantify uncertainty by producing the appropriated confidence bounds, thus pace the way for further upgrading of the acceptable level.

For the purpose of providing a firm understanding of this study, the following sections attempt (i) to provide an overview of the state-of-the art in the field of ship survivability assessment, (ii) to highlight the significance of this study by pointing out the encountered difficulties, (iii) to demonstrate the new approach with the achieved results, and (iv) to propose the research framework for the future steps on this methodology.

STATE-OF-THE ART METHODOLOGY

The most recent approaches of addressing ship survivability assessment consist of probabilistic calculations (IMO, 2009), analytical model estimation, time-domain numerical simulations and physical model testing. It appears that the reliability of the acquired prediction is proportional to the effort and cost that goes into the adopted methods, which intend to shed light on an unknown phenomenon. For instance, advanced first principle means, i.e. model experiments and numerical simulations, have been favoured for such a comprehensive assessment in general. However, the associated cost in the process should be considered at all times, a compromise must be reached between accuracy and practicality.

Analytical time to capsize (ttc) model

In response to the issue mentioned above, a desired optimal "fast and accurate" analytical model **UGD**, (Jasionowski, 2006), has been put forward as an alternative. The application of such method is intermediate between regulation computation and highly complex approaches. A great strength of this approach is to quantify the "overall" vulnerability of the ship following collision damage through a probability distribution of the time to capsize in given conditions (Figure 1). Input information is highly pertinent to the s_{ij} factor for a specific damage case and loading.

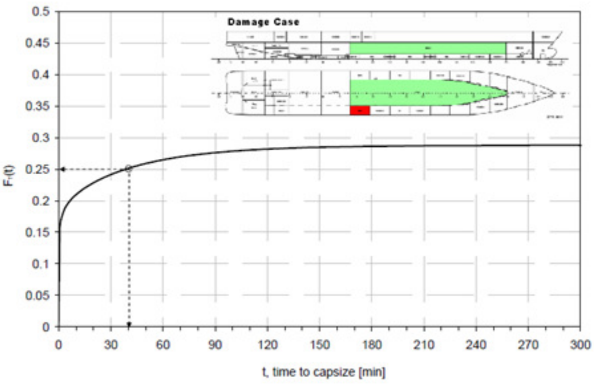

Figure 1 – Cumulative distribution of probability for time to capsize after flooding for a specific damage extent, loading and environment conditions, e.g., F_T (40min) = 25%, (Jasionowski et al, 2007)

Therefore, assuming that such time-based mathematical model is efficient to approximate the reality after verifications by relevant physical observations, from a ship design point of view, designers could predict whether the ship survives for sufficiently long time (e.g. 3 hours), to allow safe and orderly evacuation of passengers and crew. Moreover, from operational point of view, development of a decision support system for onboard application is the last chance to remedy the undesirable threats. In such circumstances, the intuitive interest lies on the evaluation of the ship's vulnerability to flooding for either imaginary or real flooding case considered as an emergency situation. The proposed analytical model enables a probability distribution of the rate of capsize for the sampled collision damage, as demonstrated in Figure 2. The derived theoretical distribution of the cumulative probability of capsize $F_T(t_{cap}|Hs)$ for any value of time (t_{cap}), together with the appropriate confidence bounds, which provides a good way to establish an acceptable level of accuracy of the model by quantifying its inherent uncertainty. The upper bound can be viewed as conservative to afford higher confidence with which the results can be used for decision making.

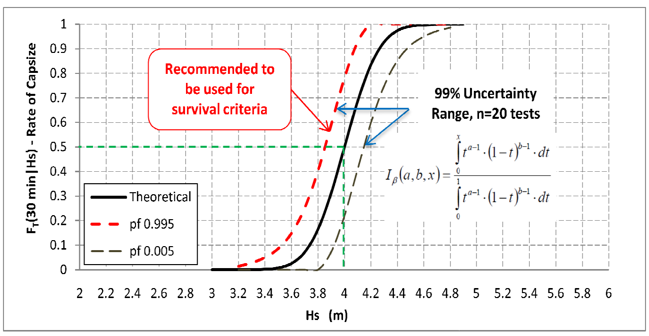

Figure 2 – Rate of capsizing, p_f, for considered sea states and given time t = 30 min.

The underlying concept behind this analytical model for addressing the cumulative probability distribution function of time to capsize is formulated in equation (1):

$$F_T(t) = \int_0^t d\tau \cdot f_T(\tau) = \sum_i^3 \sum_j^{n_{flood}} \sum_k^{n_{Hs}} w_i \cdot p_j \cdot e_k \cdot \left(1 - \varepsilon_{i,j,k}^{\frac{t}{t_0}}\right) \quad (1)$$

The terms w_i and p_j are the probability mass functions of the three specific loading conditions and n_{flood} number of flooding extents, respectively. The term e_k is the probability mass function for the sea state Hs_k, where $0 < Hs_k \leq 4$m, and n_{Hs} is the number of sea states under consideration. The term $1 - \varepsilon_{i,j,k}^{\frac{t_{cap}}{t_0}}$ represents the cumulative probability distribution of time to capsize in given conditions, that employs the theory of a Bernoulli trial process, under the assumptions that the probability of capsize p_f is constant for a given set of ship and sea state parameters. Hence, the probability that the n^{th} test is a "success" with constant probability of success p_f occurrence in each test can be obtained by equation (2), ("success" means ship capsize):

$$C = 1 - (1 - p_f)^n \quad (2)$$

In general, the period of an experiment lasting 30 minutes, $t_0 = 30\,min$, during which capsize is to be observed. The number of trails can be determined from $n = \frac{t_{cap}}{30min}$, where t_{cap} (in minutes) is considered as the cumulative amount of time necessary for ship capsizing. Thus the probability of capsize within t_{cap} / t_0 number of tests can be re-written as (3):

$$G_{T_{cap}}(t_{cap}) = 1 - (1 - p_f)^{\frac{t_{cap}}{t_0}} = 1 - \varepsilon_{i,j,k}^{\frac{t_{cap}}{t_0}} \quad (3)$$

Explicitly this is the only part that contributes to the measurement of ship vulnerability to flooding as shown in equation (1), $F_T(t_{cap})$, for a sample flooding case in the sea states under consideration. Based on the illustration in equation (3), the input information related to the rate of capsizing p_f, which is depicted by the cumulative normal distribution as demonstrated in figure 2, is presented in equation (4):

$$p_f(Hs) = \Phi\left(\frac{Hs - Hs_{crit}(s_{ij})}{\sigma_r\left(Hs_{crit}(s_{ij})\right)}\right) \quad (4)$$

Where the mean value is expressed by the critical sea state Hs_{crit} (s) as proposed by equation (5), the s_{ij} is the probability of survival, calculated according to the new probabilistic damaged stability regulation for dry cargo and passenger ships (SOLAS 2009):

$$Hs_{crit}(s) = \frac{0.16 - \ln(-\ln(s))}{1.2}, where\ p_f = 0.5 \quad (5)$$

The stand deviation is derived from an approximated capsize band width according to the following equation:

$$\sigma_r(Hs_{crit}) = 0.039 \cdot Hs_{crit} + 0.049 \quad (6)$$

In the course of the description of the analytical ttc model, the question of the validity of the predicted ship vulnerability to flooding derived from this mathematical means has been raised. A verification study has been performed on the basis of the first principle method, here it stresses on a series of physical model tests for characterizing the stochastic nature of ship capsizing after hull breach. The answer is detailed in the subsequent section.

Benchmark data on time to capsize

An extensive series of specialized physical model tests on survivability of RoPax vessels have been undertaken by pertinent projects partially funded by EC (HARDER, SAFEDOR, FLOODSTAND), and more recently by the European Maritime Safety Agency. All these tests collectively provide benchmark data for validation of the techniques on the prediction of survival time following collision damage occurrence. All model tests are carried out in accordance with the Directive 2003/25/EC. (EC, 2003)

The key objective of such dedicated tests is 1) to identify the boundary of sea states that specifying the variation of rate of capsizing (p_f) spreads from 0 to 1, 2) to produce the scatter diagram for expressing the frequency distribution of p_f for the considered sea states, and 3) to quantify the time to capsize t_{cap} (time available for evacuation) for each test run. Ultimately, the assembled data to date would be deployed in a validation exercise of the analytical ttc model which is introduced in the preceding section. Testing records obtained from SAFEDOR and FLOODSTAND projects are used as case study for the needs of this paper.

Vessel 1 – Pentalina (SAFEDOR)

A model of the RoPax vessel – Pentalina was constructed in scale 1:27, with one compartment flooded (regarded as the worst SOLAS damage) below the car deck (Figure 3) with the opening on the starboard side. The general hydrostatics and stability information in intact and damage conditions respectively are summarised in Table 1, (Chen et al, 2009).

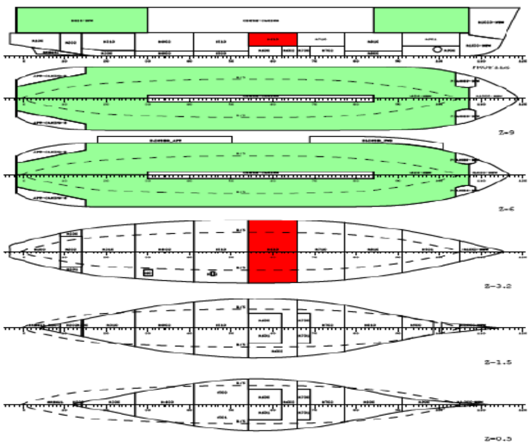

Figure 3 – Pentalina, 1-compartment damage used for model testing

Table 1 – Pentalina, Hydrostatics and stability information

Damage Case	Intact condition				Damaged stability		
	Permeability	Draught	Trim	KG	GZrange	GZmax	Heel
D5.DBS	0.95	3.6 m	0 m	5.146 m	25 deg	0.24 m	1 deg to stb

The survivability tests were completed in beam seas conditions and allowed the damaged model for free drifting. Approximately 10 successive sea states (*Hs*) between 1.5 m and 2.5 m have been measured, with an interval of 0.1m. Each of which was observed by at least 20 different time realizations. As a result, a clear range of sea states illustrating occurrence of 100% survival and 100% capsizing could be achieved. The testing data is summarized in Table 2, with a scatter diagram to represent the discrete distribution of rate of capsizing (p_f) against each target sea state in Figure 4.

Table 2 – Pentalina, Experimental test matrix

Theoretical Hs	No. of Capsize	No. of tests	Rate of Capsize
1.5	0	20	0
1.7	1	20	0.05
1.8	9	28	0.32
1.9	5	20	0.25
2	12	30	0.4
2.1	**25**	**50**	**0.5**
2.15	17	20	0.85
2.2	20	20	1.0
2.3	18	20	0.9
2.5	20	20	1.0

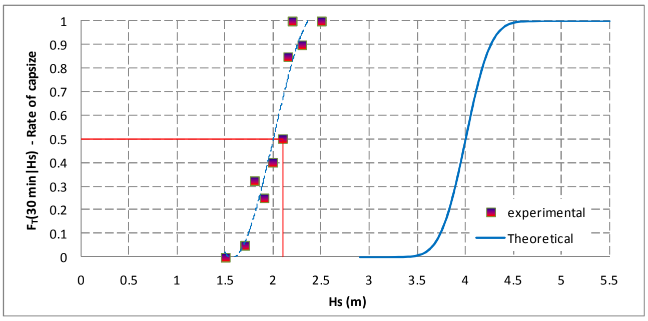

Figure 4 – Pentalina, rate of capsizing within 30 minutes based on model tests

Based on the experimental results depicted in Table 2, the measured Hs_{crit} = 2.1m where p_f = 0.5 has been selected to study the time to capsize for the next step. Following along with 50 independent survivability tests, the histogram of time to capsize (denoted as *f*) and the corresponding Cumulative Distribution Function (CDF) with 99% confidence interval are presented in Figure 5.

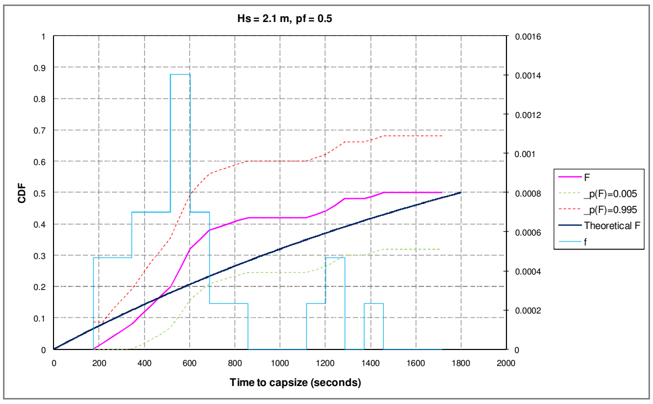

Figure 5 – Pentalina, cumulative distribution of probability for the time to capsize (F) given a specific loading, flooding extent, and sea environment, within the evaluated time = 30 minutes.

Vessel 2 - MV. Estonia (FLOODSTAND)

In the same way, other benchmarking tests provide more data that can be used for such verification work. EU project FLOODSTAND delivered a set of similar physical model experiments, which has been carried out at SSPA Sweden AB (http://www.sspa.se/). A model of the RoPax vessel (M/V Estonia) in scale 1:40 was used. A 2-compartment damage on the port side was modelled, as shown in Figure 6, (Rask, 2010). Some correlated hydrostatics and stability information is shown in Table 3. Table 4 and Figures 7, 8 represent the experimental test outcomes. Wave height of 2.6 m, where $p_f = 0.65$, is considered for establishing the CDF for the time to capsize.

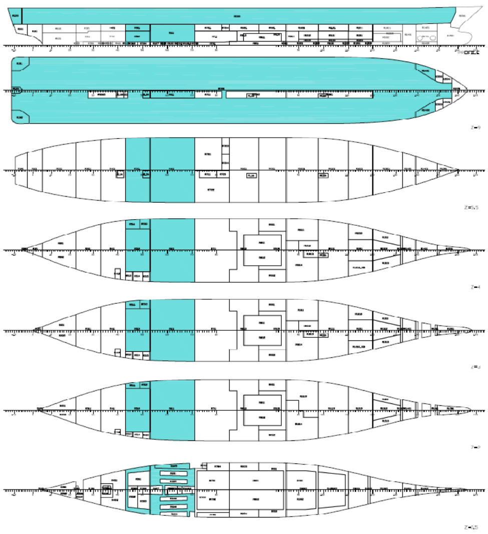

Figure 6 – M/V Estonia, 2-compartment damage

Table 3 – M/V Estonia, hydrostatics and stability information

Damage Case	Intact condition				Damaged stability		
	Permeability	Draught	Trim	KG	GZrange	GZmax	Heel
DS/P6-7.1.0	0.95	5.39 m	0 m	10.62 m	8.7 deg	0.078 m	2.3 deg to port

Table 4 – M/V Estonia, experimental test matrix

Theoretical Hs	No. of Capsize	No. of tests	Rate of Capsize
2	0	3	0
2.5	2	20	0.10
2.6	13	20	0.65
2.75	16	20	0.80
3	20	20	1

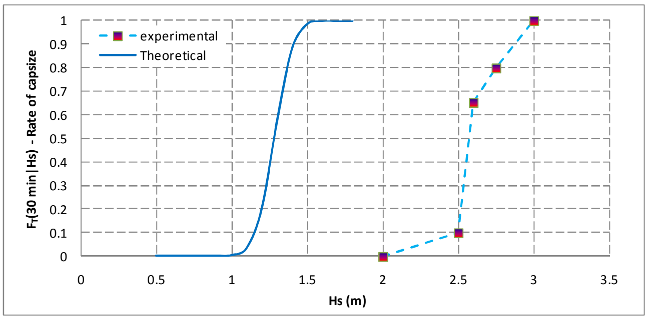

Figure 7 – M/V Estonia, rate of capsizing within 30 minutes based on model tests

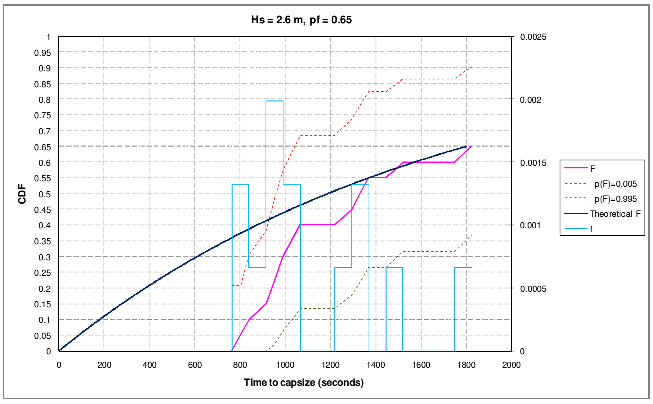

Figure 8 – M/V Estonia, cumulative distribution of probability for the time to capsize (F) given a specific loading, flooding extent, sea environment, within the evaluated time = 30 minutes.

PROBLEM REVEALED

As can be seen from Figure 4 and Figure 7, there is a notable difference between the plotted experimental results and the theoretical distributions which were computed from the exsiting analytical ttc model. The latter alternated between over and under estimation of the unknown phenomenon – rate of capsizing for the sample RoPax vessel in given conditions. In fact, the outcome explains that uncertainty is inherent in both approaches, i.e. analytical mathematical model and physical model tests. Hence, in order to identify the causes of such difficulty, the following points need to be verified in turn:

1) Some errors are present in the testing process, which may affect the recorded data to quantify the random nature of the time to capsize.

2) The proposed analytical model may need to be further improved. The input information and its sensitivity to the accuracy of input information needs to be investigated.

The uncertainties associated with the testing process were examined first. An inspection of the repeatability of the model behaviour in the same testing conditions, as well as the repeatability of the achieved sea states in comparison to the target environments have been performed. With reference to Figure 4, Hs_{crit} = 2.1m for the sample flooding case, a verification of the model capsizing at such wave height is undertaken by repeating 10 runs of survivability tests. The results are summarised in Table 5 and show that 5 cases of capsize occur out of 10 runs, i.e. p_f= 0.5, which confirms the previous determination. Moreover, a comparative experiment on 10 runs of open tank tests (without model) at Hs = 2.1m has been executed. Comparison of the obtained wave height between survivability tests and open tank tests (Table 6) in accordance with the measurements from the fixed wave probe, indicated that the actual waves generated by wave maker are higher than the targeted 2.1m. However, regarding the same time realization, the generated waves can be considered with high accuracy regardless of the presence of the model in the tank. Figure 9 provides an expression of this finding. Additionally, in order to ensure that the obtained ship behaviour is reliable, a spectral analysis of the modelled environment was performed and the area under the wave spectrum indicates that the difference between the generated wave energy and the theoretically expected one are at an acceptable level of agreement (Figure 10).

In the meantime, it is worth noting that a preliminary study of uncertainty assessment has been put forward by Cichowicz et al., (2009) in order to investigate the possible main sources of error regarding the experimental data of ship response in roll motion. Although the results do not provide answers to many important questions to date, but based on this study the broad range of problems (uncertainties) associated with model testing measurements can be reduced.

Table 5 – Verification of the critical sea state (Pentalina)

MODEL TEST EXPERIMENTS				
TARGET WAVE HEIGHT: 2.10m				
SHORT WAVES				
Run No.	Wave	Wave Probe	Capsize	
		Fixed (m)	Yes	NO
1	JSHs210R01	2.19	X	
2	JSHs210R02	2.24	X	
3	JSHs210R03	2.26		X
4	JSHs210R04	2.22	X	
5	JSHs210R05	2.29		X
6	JSHs210R06	2.32		X
7	JSHs210R07	2.29		X
8	JSHs210R08	2.24	X	
9	JSHs210R09	2.34	X	
10	JSHs210R10	2.32		X

Table6 – Open tank tests for 10 different time realizations at Hs = 2.1m (Pentalina)

MODEL TEST EXPERIMENTS		
TARGET WAVE HEIGHT: 2.10 m		
OPEN TANK TEST (Without Model)		
Run No.	Wave	Wave Probe
		Fixed (m)
1	JSHs210R01	2.33
2	JSHs210R02	2.27
3	JSHs210R03	2.27
4	JSHs210R04	2.28
5	JSHs210R05	2.26
6	JSHs210R06	2.26
7	JSHs210R07	2.29
8	JSHs210R08	2.35
9	JSHs210R09	2.33
10	JSHs210R10	2.31

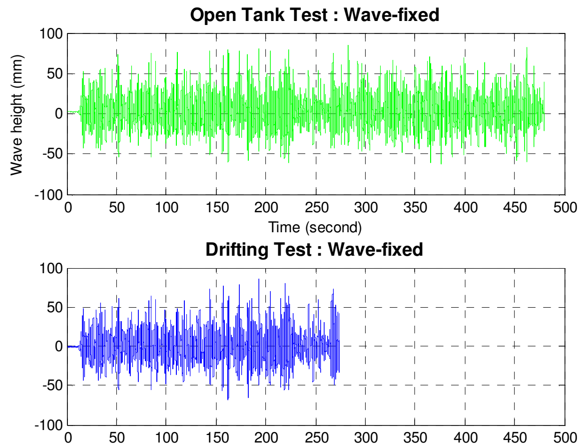

Figure 9 – Time domain wave form for the case of capsizing (Refer to Table 5, wave JS210HsR02)

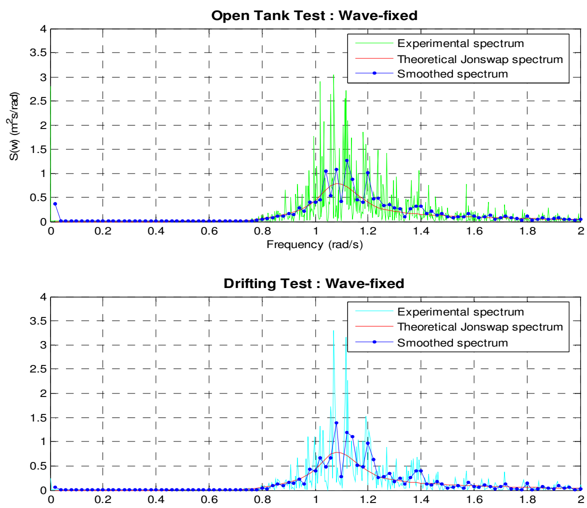

Figure 10 – Frequency domain wave spectrum for the capsizing case (JS210HsR02)

On the other hand, attention should be paid on the analytical model itself. Concerning the limitations of this approach, the proposed prediction of the rate of capsizing (p_f) follows the normal distribution as expressed in equation (4), where uncertainty is directly linked with the mean (defined as the critical Hs) and standard deviation of the model. An obvious diffifultly is to work out the mean value, which is associated with the "$s_{i,j}$" factor for each specific damage case and loading, according to equation (5). Thereby a common approval for the equation of "s" is a prerequisite for getting the theoretical critical sea state Hs_{crit}. However, it has not been confirmed yet especially for RoPax vessels. Furthermore, additional complexity related to the derivation of the standard deviation according to the existing experimenal data. Currently, a linear equation containing Hs_{crit} has been used, equation (6), to deal with this factor, which was developed from a regression of the capsize bandwidth, (Jasionowski, 2006). Such distance (Figure 11) between $\mu \pm 2.5758\sigma$ specifies that there is 99% confidence that the variation of the critical sea state where $p_f = 0.5$ spreads within this band exactly. However, the approximated linear expression of capsize bandwidth ($2 \times 2.5758\sigma$) can not be brought into wide use for the entire group of intertested vessels or damage scenarios, as the size of test matrix has been limited for practical purposes. So it becomes obvious that such an analytical model for survivablity assemssemt has substantial margin for further improvement. Until then, making a good inference about the equation of interest on the basis of the observed data could afford a new surge of thinking that tackles the identified gap between the analytical model and the model test results. Therefore, the manner in which the available test data can be used to represent reality will be the focal point of the next chapter.

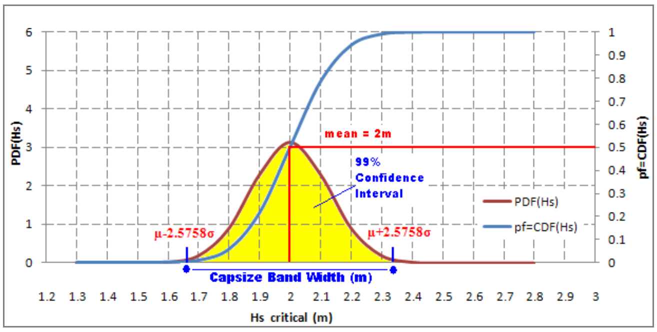

Figure 11 – The concept of a capsize band

A NEW APPROACH THROUGH INFERENCE

This approach builds on recorded experimental data and intends to offer prediction of the ship vulnerability in a specific sea environment within a given time after flooding has commenced. The obtained data is the core of such approach, and has definitive effect on the outcome. The traditional way to make an approximation of the observation by assigning a regression curve is inefficient in this case. The following sections endeavour to introduce the major algorithms applied in this technique in detail.

Generalized Linear Model (GLM) for Binary Data

In the knowledge that the essential activity is virtually to model relationships between several physical variables (loading conditions, flooding extent, environmental conditions) and the ship response when subject to flooding damage, some sort of regression analysis is needed. Each single observation of the status of the ship (survival or capsizing) has one of the two outcomes, denoted by 0 and 1. Thus the classical regression analysis is not suitable as the dependent variable (i.e. ship behaviour) is a typical binary parameter. A more sophisticated method for analyzing binary response data is needed. In practice, one of the original models, known as **probit model,** (Dobson & Barnett, 2008), has been identified as a promising solution in addressing such difficulty.

The probit model was developed by Bliss, (1934), in an attempt to study the relationship between the dosage and the mortality. Responses of such model were the proportions (or percentages) of 'success' π, as a function of the dosage level, x. For a randomly selected subject, let the binary response variable, $Y = 1$ if the subject dies, or $Y=0$ indicates the subject survives. For a fixed dosage x, the probability of a randomly selected subject success (death) is:

$$\pi(x) = P(Y = 1|X = x) = \Phi(\alpha + \beta x) \qquad (7)$$

Let Φ denotes the cumulative probability function for the standard Normal distribution N (0, 1). Thus in GLM form,

$$\Phi^{-1}[\pi(x)] = \alpha + \beta x \qquad (8)$$

is the **probit model**.

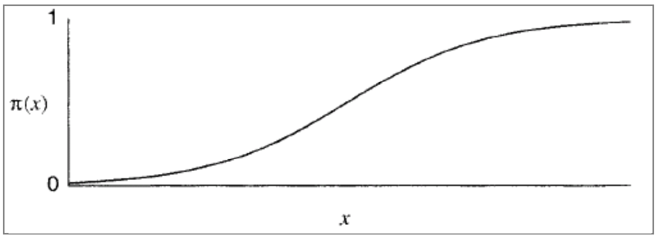

Figure 12 – A simple probit model

Deployment of the probit model offers the following benefits:

- The response curve for $\pi(x)$ has the appearance of the normal CDF with mean $\mu = -\alpha/\beta$ and standard deviation $\sigma = 1/|\beta|$, as shown in Figure 12. So shapes of different CDF occur as α and β vary. Replacing x by βx permits the curve $\pi(x)$ to increse at a different rate than the standard CDF (or even to decrease if β<0); varying α moves the curve to the left or right. This flexibility offers a unique platform for presenting the response variable, "ship behaviour", probabilistically, which is in line with the exiting technique of getting the rate of capsizing p_f, equation (4).

- The model can be extended to include more than one independent variable at a time by adding more influencing terms, e. g $\alpha + \beta_1 x_1 + \cdots + \beta_n x_n$. Although current interest lies mainly on the establishment of the correlation between $p(capsize|defined\_conditions)$ and $Hs_{critical}$, the model can cater more influencing variables simultaneously, e.g. Gz_{max}, $Range$, etc.

Bayesian Regression

Having identified the mathematical model, the relationship will be established once the coefficients α, β_i can be estimated. Classic solving techniques, such as *Ordinary Least Square* for standard linear model, *Fisher scoring algorithm for Generalised Linear Model*, and *Newton-Raphson's Maximum Likelihood estimation* techniques have received wide recognition. Nevertheless, those methods may perform poorly for dealing with small samples or models with many parameters. Thereby, more attention is being paid to *Statistical Inference* from a *Bayesian perspective*, (Gelman, 2004), which uses probabilities to represent a set of rational beliefs about unknown model parameters. Bayes' theorem, equation (9), provides a rational method for updating beliefs in light of new evidence from the data. In practice, Bayesian inference is a robust technique to estimate the probability of the event happening regardless of the sampling size.

$$p(\theta|y) = \frac{p(\theta)p(y|\theta)}{p(y)} \quad (9)$$

Bayesian Inference

In accordance with Bayes' theorem, the physical meaning can be derived as: if θ denotes the unobserved population parameters of interest, *y* denotes the observable data. The denominator is usually not computed since it is not a function of θ. Therefore, the posterior probability $p(\theta|y)$is proportional to the product of the prior information, $p(\theta)$, and the collected evidence from the data, $p(y|\theta)$. As it can be seen, Bayesian inference combines the information from the observed data and prior beliefs. In this case, as each new observation is obtained, the posterior distribution is updated by treating the previous posterior as the prior beliefs. Apparently this iterative computation process is complicated, and in fact, such difficulty has been overcome to a large extent in the last 10 – 15 years due to advances in integration methods, particularly, *Markov Chain - Monte Carlo* (MCMC) method. The most common criticism in Bayesian Statistics is how to choose the prior, since the prior density of θ is user-defined and reflects the user's believes, and therefore it is subjective. However, sometimes the prior probability is required to reflect a situation when there is a complete lack of knowledge about the interested parameter. For this case, MCMC algorithms are very attractive in summarizing posterior distribution, as they are easy to set up and program, especially require relatively little prior input from the users. Some detailed description is left in the following part.

MCMC algorithm - Gibbs Sampling

Using Bayesian inference for multiple parameter problems, for instance the mean and the variance which are the most two classic applied measures for describing the data set, hence, the equation (9) can be transformed into:

$$p(\mu, \sigma^2|y_1, \dots, y_n) = \frac{p(\mu, \sigma^2)p(y_1, \dots, y_n|\mu, \sigma^2)}{p(y_1, \dots, y_n)} = \frac{p(\mu|\sigma^2)p(\sigma^2)p(y_1, \dots, y_n|\mu, \sigma^2)}{p(y_1, \dots, y_n)} \quad (10)$$

In this particular case, more than one parameter needs to be approximated simultaneously. To do so, the Markov Chain Monte Carlo simulation can be employed. It is a general method based on drawing values from approximate distributions and then correcting those draws to better approximate the target posterior distribution. The samples are drawn sequentially, with the distribution of the sampled draws depending on the last value drawn. The draws form a Markov chain. The process follows mainly two algorithms: the *Gibbs sampling* and the *Metropolis-Hasting*.

In this section, we emphasize on the use of Gibbs sampling algorithms to provide an introduction of setting up an MCMC algorithm in summarizing posterior distribution. Referring to equation (10), the joint posterior probability distribution $p(\mu, \sigma^2|y_1, \dots, y_n)$ can be achieved by iteratively sampling the full conditional distribution of μ and σ^2 respectively, e.g. $p\left(\mu|y_1, \dots, y_n, \sigma^2\right), p\left(\sigma^2|y_1, \dots, y_n, \mu\right)$ as each one is a conditional distribution of a parameter given everything else. At each iteration, assume it stands at a state with parameters $\phi^{(s)} = \left\{\mu^{(s)}, \sigma^{2(s)}\right\}$, a new state will be:

1) Sample $\mu^{(s+1)} \sim p\left(\mu|y_1, \dots, y_n, \sigma^{2(s)}\right)$
2) Sample $\sigma^{2(s+1)} \sim p\left(\sigma^2|y_1, \dots, y_n, \mu^{(s+1)}\right)$
3) Let $\phi^{(s+1)} = \left\{\mu^{(s+1)}, \sigma^{2(s+1)}\right\}$

The resultant sample will be a dependent sequence of vectors for both μ and σ^2: $\phi^{(1)}, \phi^{(2)}, \dots, \phi^{(N)}$. In this sequence, $\phi^{(s+1)}$ depends on $\phi^{(1)}, \phi^{(2)}, \dots, \phi^{(N)}$ only through $\phi^{(s)}$.

In the case of probit model training, identical principal applies except that the interested variables will be $\left\{\alpha, \beta_1, \beta_2, \dots, \beta_N, \sigma^2\right\}$. The normal treatment would be to establish a matrix containing all the model coefficients, $\beta = \left[\alpha, \beta_1, \beta_2, \dots, \beta_N\right]$. Hence, by iteratively sampling and updating β and σ^2 from their corresponding full conditional distribution, the distribution for each coefficient can be obtained.

Confidence interval estimation

With Bayesian inference, the probit model can be constructed through the posterior distributions of estimated model coefficients (i.e. α, β_i). Hence, for each value of the independent variables (i.e. *Hs*), there is a corresponding probability distribution of the dependent response variable (i.e. ship behaviour). As a result, the establishment of the confidence interval of the derived response regarding each independent variable value is just a matter of quantile estimation.

Preliminary Results

Based on the explanation of the fundamental ideas behind this new approach, this section demonstrates the preliminary results from an application of binary regression with a probit link. Figures 13 and 14 reveal the new distributions of the rate of capsizing, from which the developed results are contrary to the theoretical distribution derived from the analytical ttc model. The displayed new Bayesian distribution, cooperated with an estimation of the 99% confidence intervals, present an excellent fit to the experimental results since there is an elimination of either over or under estimation. Thus the proposed new approach can be considered as a promising alternative to express ship behaviour with higher confidence based only on evidence of observed data. Modern Bayesian computing can be considered as a robust manner to process data analysis.

FURTHER WORK

In the long run, more effort should be invested in two aspects:

1) Thorough investigation of the preliminary results. A review of the assisted algorithms is necessary.

2) The practical application of the research results should be treated carefully.

Refining the probit model

As discussed in the foregoing, one of the strong points for employing the probit model is attributed to its ability to incorporate multiple independent variables together, i.e. (x_1, x_2,...,x_n). This means that dominant variables affecting ship survivability can be considered simultaneously. So far, only single variable, sea state (Hs_{crit}), has been included in the probit link to create the distribution of $\pi(x)$ as the probability density of the rate of the capsize $p(capsize|Hs)$

$$\Phi^{-1}[\pi(x)] = \alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n \qquad (11)$$

In order to get proper distributions that reflect reality better, it is desirable to develop a mathematical model that consists of more influencing variables for characterizing ship survivability for a given damage of a specific design. This could be achieved by including Gz_{max}, $Range, etc.$, of a damage condition. Therefore, an evolved form of this model (see equation (11)) is the generalized application for dealing with each damage scenario independently.

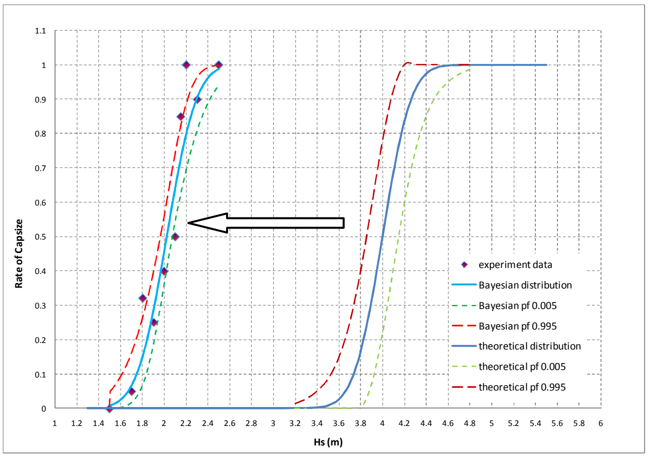

Figure 13 – Pentalina, new distribution of rate of capsizing simulated by Bayesian techniques

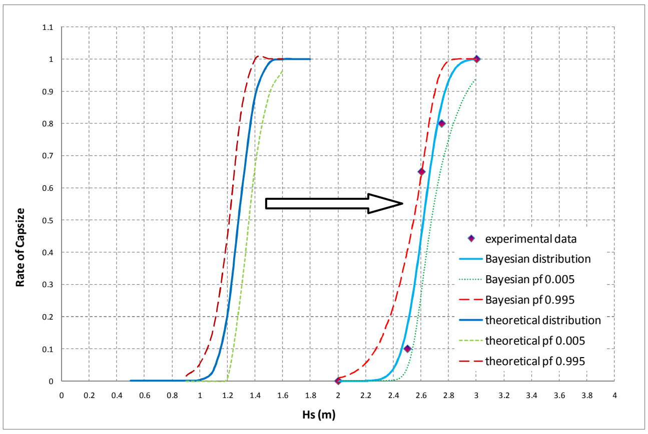

Figure 14 – M/V Estonia, new distribution of rate of capsize simulated by Bayesian techniques

Review of MCMC algorithms

It is easy to see that the applied generalized linear model (probit link function) include more than one parameter. Generally, for many multi-parameter models the joint posterior distribution is nonstandard and difficult to sample directly. However, this can be overcome by making sample from the full conditional distribution of each parameter, $p\left(\alpha \mid \beta_1, \ldots, \beta_n, \sigma^2\right)$. An iterative MCMC algorithm as Gibbs sampling has been used to approximate the posterior distribution. However, the precondition of such application is when the prior distribution is available or desirable (informative), otherwise the full conditional distributions of the parameters do not have a standard form and the Gibbs sampler cannot be easily used. In such situation, the Metropolis-Hastings algorithm is regarded as an alternative of approximating the posterior distribution corresponding to any combination of prior distribution and sampling model. Furthermore, for the non-normal generalized linear mixed models, a Metropolis-Gibbs algorithm has been suggested to summarize the posterior distribution of the parameters in common. For this reason, an in-depth study of MCMC algorithms applied in Bayesian inference is needed.

Implementation of new achievements

Once the probit model can be trained and the new method dealing with the survivablity assessment is completed, the output $p(capsize|Hs)$ will be added to the equation (3) for survival time assessment. The ultimate result can be summarised by a CDF curve for time to capsize in a given damage condition showing a process to compute the ship vulnerability to flooding. In order to increase the argreement of the results from such probablilic study, quantifying its uncertainty is an additional task to be undertaken. Thus the methods of setting up confidence intervals should be examined. In the end, a higher confiderce with the achieved results can be used to form a time-based survival criteria for assisting the decision support system on board in any emergency situation.

On the other hand, through the latest research, the new harmonised probabilistic rules addressing damaged stability (SOLAS 2009) has been pointed out that they are possible to create ship designs with significant deficits in safety for passenger Ro-Ro vessels, (HSVA, 2009). Hence, additional effort should be pursued for affording the most appropriate solution to improve current stability standards. The existing anlytical time to capsize model can be regarded as a rational basis to put forward the recommendation on amendment of the SOLAS 2009 damage stability rules, because such model applies a correlation between the developed "s" factor in rules and the input variable Hs_{crit}, as depicted in equation (5). Therefore, the achieved new distribution, $p(capsize|Hs)$ can be used to formulate a new connection between the rate of capizing and the critical sea state. In turn, the computed Hs_{crit} may reversely affect the "s" factor eventually. Due to the new assessment technique presented in this paper is close related to a comprehensive study of physical stochastic nature of ship capsizes, for this reason some complex underlying physical phenomena as an instance of multiple effects of free surface on vehicle deck, as well as different ship configurations have been implicitly accommodated in the trained model. As a result, the achieved distribution of time-based survival criteria may support a 'sufficient' level of safety facilitated by SOLAS 2009 for passenger Ro-Ro vessels.

CONCLUSIONS

In light of the need to further refine the state-of-the-art techniques for addressing ship vulnerability to flooding through a time to capsize model, this paper focuses on the establishment of a new methodology for modelling the stochastic behaviour of time to capsize with particular emphasis on the underlying principles and its advantages. The preliminary results obtained are promising, however, further investigation of both data analysis techniques and the ensuing applications are needed. It is believed that this research has positively contributed towards ship survivability assessment.

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